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Bianchi type-V Cosmological model with *G* and Λ in Scale Covariant Theory of Gravitation

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Abstract: **In this paper, we have investigated some features of anisotropic accelerating Bianchi type-V cosmological models in the presence of perfect fluid, variable gravitational and cosmological constants towards the gravitational field equations for scale covariant theory of Gravitation. With the help of special law of variation of Hubble's parameter proposed by Berman (1983) which yields constant deceleration parameter of the models, we achieve a physically realistic solution to the field equations. Some geometrical and physical aspects of derived model are discussed with the help of solution and make the models at late times turn out to be flat Universe.**

Keywords: **Bianchi type-V cosmological model; scale covariant theory of gravitation.**

1. INTRODUCTION

Since, the Einstein field equation has two parameters; one is the gravitational constant *G* and second is the cosmological constant Λ. As, the Newtonian constant of gravitation *G* plays the role of a coupling constant between geometry and matter in the Einstein field equation. In an evolving universe, it appears to look at this constant as a function of time. There are significant observational evidence that the expansion of the Universe is undergoing a late time acceleration [1–5]. This, in other words, amounts to saying that in the context of Einstein's General Theory of Relativity some sort of dark energy, constant or that varies only slowly with time and space dominates the current composition of cosmos.

Among many possible alternatives, the simplest and most theoretically appealing possibility for dark energy is the energy density stored on the vacuum state of all existing fields in the universe, i.e., $\rho = \frac{\Lambda}{8\pi G}$ where Λ is the cosmological constant. However, a constant Λ cannot explain the huge difference between the cosmological constant inferred from observation and the vacuum energy density resulting from quantum field theories. In an attempt to solve this problem, variable Λ was introduced such that Λ was large in the early universeand then decayed with evolution. Cosmological scenarios with a time-varying Λ were proposed by several researchers such as Vishwakarma [6], Cunha and Santos [7], Carneiro and Lima [8]. A modification linking the variation of *G* with that of variable Λ term has been considered within the framework of General Relativity by a number of workers such as Berman [9] Kallingas *et al.* [10]. This modification is appealing as it leaves the form of Einstein's equations formally unchanged by allowing a variation of *G* to be accompanied by a change in Λ. Cosmological models with time-dependent *G* and Λ in the solutions $\Lambda \approx R^2$, $\Lambda \approx t^{-2}$ were first obtained by Bertolami [11]. The cosmological models with variable *G* and Λ have been studied by several authors like Arbab [12], Sattar and Vishwakarma [13], Pradhan *et al.* [14-16], Singh *et al.* [17, 18].

The Scale-Covariant Theory of Gravitation by associating the mathematical operation of scale transformation with the physics of using different dynamical systems to measure space-time distances [19], also, which provides the necessary theoretical framework to logically talk about the possible variation of the gravitational constant *G* without compromising the validity of General Relativity. In this theory, we measure physical quantities in atomic units whereas Einstein's field equations in gravitational units. The metric tensor in the two systems of units are related by a conformal transformation,

$$\bar{g}_{ij} = \phi^2 g_{ij}, \tag{1}$$

where the metric $\bar{g}_{ij}$ giving macroscopic metric properties and g_{ij} giving microscopic metric properties. Here we consider the gauge function ϕ as a function of time.

The general Einstein's field equations by using the conformal transformations equations (in scale-covariant theory of gravitation) is,

$$R_{ij} - \frac{1}{2} g_{ij} R + f_{ij}(\phi) = -8\pi G T_{ij} + \Lambda g_{ij}, \tag{2}$$

where

$$\phi^2 f_{ij}(\phi) = 2\phi\phi_{i;j} - 4\phi_i\phi_j - g_{ij}(\phi\phi^{\lambda}_{;\lambda} - \phi^{\lambda}\phi_{\lambda}). \tag{3}$$

Here comma denotes ordinary partial differentiation whereas a semi-colon denotes a covariant differentiation.

2. FIELD EQUATIONS, METRIC AND GENERAL EXPRESSIONS

We consider the line element for an anisotropic Bianchi type-V space-time is of the form as

$$ds^2 = dt^2 - A^2 dx^2 - e^{2mx} B^2 dy^2 - e^{2mx} C^2 dz^2, \tag{4}$$

where the scale factors *A*, *B*, *C* being functions of cosmic time only and*m* be the any arbitrary constants.

The source ofgravitational field is considered as a perfect fluid. So for aperfect fluid, the energy momentum tensor is given by

$$T_{ij} = (p + \rho) u_i u_j - p g_{ij}, \tag{5}$$

where ρ is the energy-density, p the pressure and u^i is the four velocity vector of the fluid following $u^i u_j = 1$.

The general formulas of certain physical parameters for the metric equation (4) are given as follows:

The average scale factor and spatial volume respectively as

$$R = (ABC)^{1/3}, \qquad V = ABC \tag{6}$$

The generalized mean Hubble parameter which expresses the expansion rate of the space-time, can be given as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{7}$$

where H_1, H_2, H_3 are the directional Hubble parameter in the direction of x, y, and *z*-axis respectively.

It should be noted that the parameters *H*, *V* and *R* are connected by the following relation

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{R}}{R}, \tag{8}$$

To discussed whether the models either approach isotropy or not, we define an anisotropy parameter of the expansion as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{9}$$

where $H=(\ln R)^{\cdot}=\frac{\dot{R}}{R}=\frac{1}{3}(H_1+H_2+H_3)$, $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}$, $H_3=\frac{\dot{C}}{C}$ are the directional Hubble parameters of *x*, *y* and *z*-axes respectively.

The expansion scalar and shear scalar respectively are defined as follows

$$\theta=u^{\mu}_{;\mu}=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}, \tag{10}$$

$$\sigma^2=\frac{3}{2}H^2A_m. \tag{11}$$

Another important dimensionless kinematic quantity is the deceleration parameter, which showswhether the universe exhibits accelerating volumetric expansion or not:

$$q=-1+\frac{d}{dH}\left(\frac{1}{H}\right), \tag{12}$$

For $-1\le q<0$, $q>0$ and $q=0$ the universe exhibit accelerating volumetric expansion, decelerating volumetric expansion and constant-rate volumetric expansion respectively.

The components of the field equations given in equations (2) and (3) for the metric equation (4) using the source as a perfect fluid provided in equation (5), are given as follows

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{m^2}{A^2}=(-8\pi)pG-\Lambda+2\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{\phi}}{\phi}\right)\frac{\dot{\phi}}{\phi}+2\frac{\ddot{\phi}}{\phi}, \tag{13}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{m^2}{A^2}=(-8\pi)pG-\Lambda+2\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}-\frac{\dot{\phi}}{\phi}\right)\frac{\dot{\phi}}{\phi}+2\frac{\ddot{\phi}}{\phi}, \tag{14}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{m^2}{A^2}=(-8\pi)pG-\Lambda+2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}-\frac{\dot{\phi}}{\phi}\right)\frac{\dot{\phi}}{\phi}+2\frac{\ddot{\phi}}{\phi}, \tag{15}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{3m^2}{A^2}=(8\pi)\rho G-\Lambda+2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{\phi}}{\phi}\right)\frac{\dot{\phi}}{\phi}, \tag{16}$$

$$2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}=0. \tag{17}$$

Here in what follows an over dot denotes ordinary differentiation with respect to time *t.*

The Vanishing divergence of the Einstein's tensor is

$$\left(R_{ij}-\frac{1}{2}g_{ij}R\right)_{;i}=0, \tag{18}$$

which leads to

$$8\pi\dot{G}\rho+\dot{\Lambda}+8\pi G\left[\dot{\rho}+(p+\rho)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]=0. \tag{19}$$

The law of conservation of energy is

$$\left(T^{ij}_{;j}\right)=0, \tag{20}$$

which leads to

$$\left[\dot{\rho}+(p+\rho)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]=0. \tag{21}$$

Using the above equations (19) and (21), this gives

$$\dot{G}=-\frac{\dot{\Lambda}}{8\pi\rho}, \tag{22}$$

Assume that the fluid obeys the condition of barotropic equation of state as

$$p=\gamma\rho. \tag{23}$$

Equation (17) yields

$$A^2=BC. \tag{24}$$

3. SOLUTION OF FIELD EQUATIONS

As the field equations (13)–(16) are a system of four equations with seven unknown parameters. The system is thus initiallyundetermined and we need additional constraints toclose the system. The observations of the velocityred-shift relation for extragalactic sources suggestthat Hubble expansion of the Universe is isotropictoday within ≈ 30 percent. To put it moreprecisely, red-shift studies place the limit as $\sigma/H \le 0.3$ on the ratio of shear σ to Hubble's parameter H in the neighborhood of our galaxytoday. Collin *et al*. (1980) have pointed out that forspatially homogeneous metric the normalcongruence to the homogeneous expansion satisfies the condition that σ/H is constant i.e., theexpansion scalar is proportional to the shear scalar, which gives the relation between two metric potentials as

$$A=B^{1/\alpha}, \tag{25}$$

where α be the any positive constant.

In addition, with the help of special law of variation of Hubble's parameter proposed by Berman that yields constant deceleration parameter models of the universe,

$$q=-\frac{R\ddot{R}}{\dot{R}^2}=\text{constant}. \tag{26}$$

where R be the overall scale factor. The constant is taken as negative to obtain an accelerating model of the universe.

From (26) we obtain

$$R=(at+b)^{\frac{1}{1+q}}, \tag{27}$$

where $a\neq 0$ and b are the constants of integration.

Equation (6) yields

$$(ABC)^{\frac{1}{3}}=(at+b)^{\frac{1}{1+q}}. \tag{28}$$

Using (24) in (28) we find

$$A=(at+b)^{\frac{1}{(1+q)}}, \tag{29}$$

$$B=(at+b)^{\frac{\alpha}{(1+q)}}, \tag{30}$$

$$C = (at+b)^{\frac{2-\alpha}{(1+q)}}, \tag{31}$$

Hence, using the equations from (29) to (31) the line element for an anisotropic Bianchi type-V space-time is defined as

$$ds^2 = dt^2 - (at+b)^{\frac{2}{(1+q)}} dx^2 - (at+b)^{\frac{2\alpha}{(1+q)}} e^{2mx} dy^2 - (at+b)^{\frac{2(2-\alpha)}{(1+q)}} e^{2mx} dz^2 . \tag{32}$$

The model (32) represents Bianchi type-V cosmological model with G and Λ in Scale-covariant theory of gravity with negative constant deceleration parameter. As the derived model is anisotropic with respect to constant α (for $\alpha \neq 1$ the model is anisotropic while for $\alpha = 1$ the model is isotropic) along with the model has a singularity and the singularity observed at point.Atan initial stage, all the metric potentials are constants. Hence, initially the model has no singularity but at the point $t_s = -b/a$, above equation represent a singular model. As a special case for $\alpha = 1$ our derived universe approaches isotropy.

4. PHYSICAL PARAMETERS

Energy Density

$$\rho = \frac{k}{(at+b)^{\frac{3(1+\gamma)}{(1+q)}}} . \tag{33}$$

It is observed that, the energy density obtained in the equation (33) is always positive except at a_1 and decreasing function of cosmic time. The behavior of energy density versus cosmic time is clearly shown in figure (*i*) with the appropriate choice of constants.

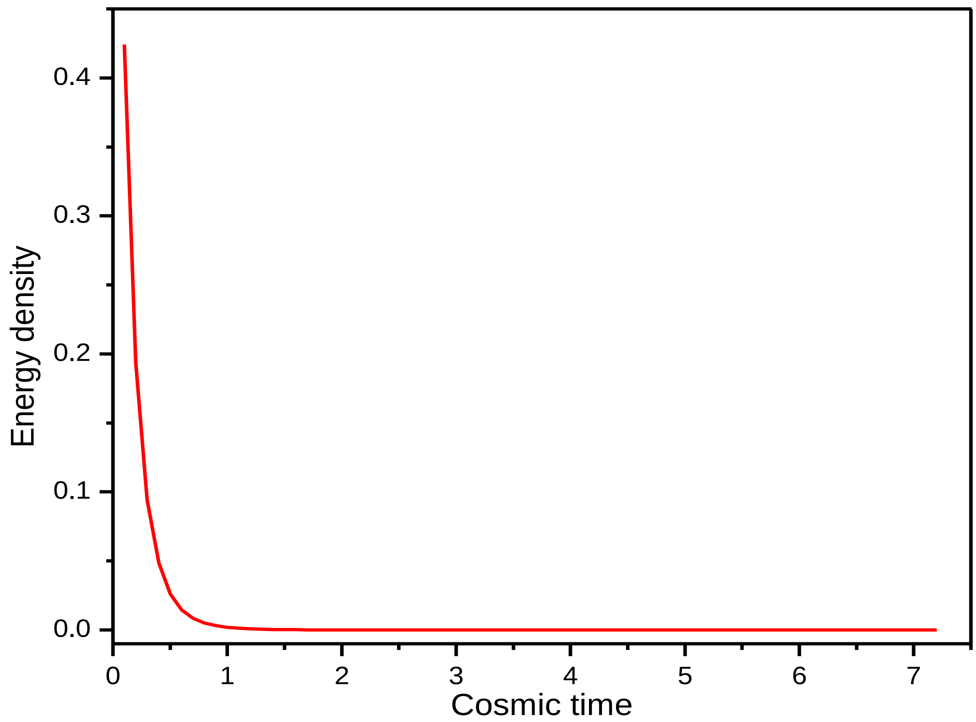

Figure (i): The behavior of energy density versus cosmic time with the appropriate choice of constants.

Cosmological Constant,

$$\Lambda = \frac{3a^2\beta}{(1+q)^2(at+b)^2} . \tag{34}$$

The cosmological constant derived in equation (34) is also always positive and decreases with cosmic time, the relation between are $\Lambda \propto {}^{1}\!/_{t^2}$ i.e. $\Lambda \propto t^{-2}$ as expected. The routine of cosmological constants versus cosmic time is clearly revealed in figure (*ii*) with the appropriate choice of constants. Hence in our investigation the behavior of Cosmological Constant resembles with the work investigated by Bertolami [11].

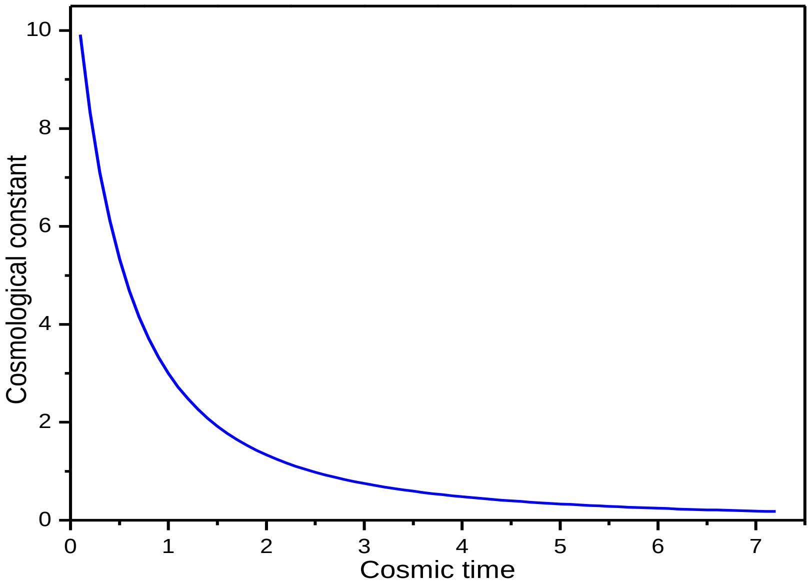

Figure (ii): The behavior of Cosmological constant versus cosmic time with the appropriate choice of constants.

As the behavior of cosmological constant is on the sign of the constants α and β, if both $\alpha > 0$ and $\beta > 0$ or $\alpha < 0$ and $\beta < 0$ both the situations provided that $\Lambda > 0$. If either $\alpha > 0$ and $\beta < 0$ or $\alpha < 0$ and $\beta > 0$ the $\Lambda < 0$.

Gravitational Constant,

$$G = \frac{3a\beta}{8\pi k(1+q)(3\gamma - 2q + 1)} (at + b)^{\frac{(3\gamma - 2q + 1)}{(1+q)}} . \tag{35}$$

The behavior gravitational constant versus cosmic time is clearly depicted in figure (*iii*) with the appropriate choice of constants.

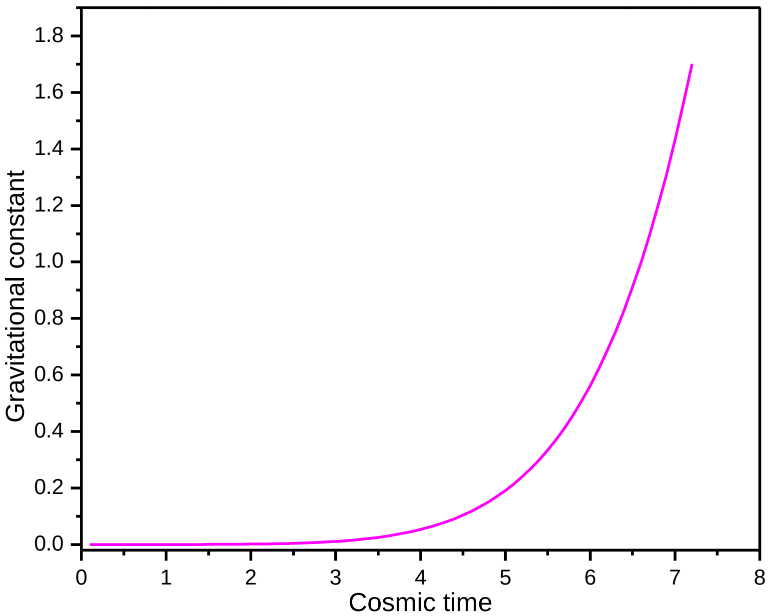

Figure (iii): The behavior of Gravitational constant versus cosmic time with the appropriate choice of constants.

As the gravitational constants is always constant, and increases with cosmic time hence $G \propto t$ as expected.

Anisotropic Pressure,

$$p = \left(\frac{-1}{8\pi G}\right)\left(\frac{a^2(2+\alpha^2-2\alpha-2q-4n+n^2-2nq+3\beta}{(1+q)^2(at+b)^2} - \frac{m^2}{(at+b)^{\frac{2}{(1+q)}}}\right). \quad (36)$$

From the equation (36), it is observe that the pressure in the derived model is always positive (which shows that there is no chance of dark energy) and decreases with respect to time (see the figure-*iv*). Hence the model is fully occupied with only barotropic fluid with radiation dominated era.

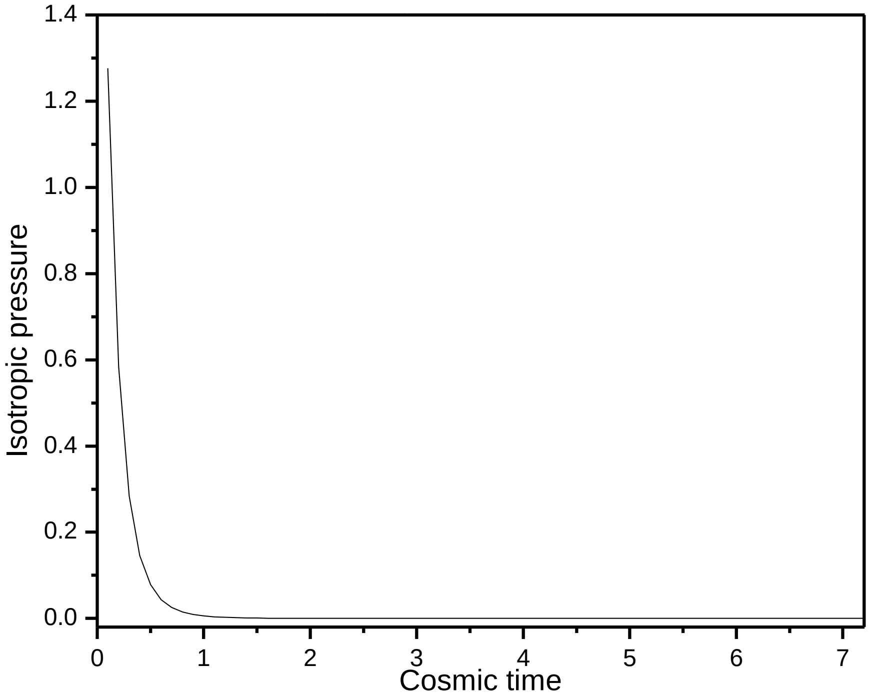

Figure (iv): The behavior of isotropic pressure versus cosmic time with the appropriate choice of constants.

5. KINEMATICAL PARAMETERS

The kinematical properties which are important in cosmology for discussing the geometrical behavior of the universe that are spatial volume, Hubble parameter, expansion scalar, shear scalar and anisotropic parameter which have the following expressions

Spatial volume

$$V = (at+b)^{3/(1+q)} e^{2mx}.$$

It is observed that the spatial volume V has constant value at an initial time t = 0, expands exponentially as t increasesand becomes infinitely large at $t = \infty$.

Hubble's parameter

$$H = \frac{a}{(1+q)(at+b)}.$$

Expansion scalar

$$\theta = \frac{3a}{(1+q)(at+b)}.$$

Shear scalar

$$\sigma^2 = \left[\frac{a(1-\alpha)}{(1+q)(at+b)} \right]^2 .$$

Anisotropy parameter

$$A_m = \frac{2}{3}(1-\alpha)^2 .$$

It is observed that the kinematical parameters such as Hubble's parameter, expansion scalar, shear scalar and anisotropic parameter all are the functions of cosmic time with decreasing behavior. Hence at initially all are attending a constant value and with the expansion all are contracted and at an infinite expansion and at a singular point all are diverge. The ratio of θ / σ tends to constant, thus the model approaches anisotropy and matter isdynamically negligible near the origin.

Deceleration parameter

$$q = -0.5 .$$

To describe the dynamics of the universe the deceleration parameter q is currently a serious candidate among all the physical quantities of interest in cosmology. According to the prediction of standard cosmology, the universe has a phase transition from decelerating to accelerating and also reveals that expansion of the universe is speeding up instead of slowing down in outlook of recent observational evidences of the high red shift of type-Ia supernova. The sign of q indicates whether the model accelerates or not. The positive sign of $q$corresponds to decelerating models whereas the negative sign indicates acceleration.

6. CONCLUSION

In the investigationof anisotropic accelerating Bianchi type-V cosmological models in the presence of perfect fluid, variable gravitational and cosmological constants towards the gravitational field equations for scale covariant theory of gravitation using special law of variation of Hubble's parameter which yields constant value of deceleration parameter, it is observed that the models at late times and for $\alpha = 0$ turn out to be flat Universe.The energy density is always positive and decreasing function of cosmic time except for $a_1 > 0$. Along with the cosmological constant also shows the same behavior as that of energy density i.e. always positive and decreases with cosmic time and the relation between them is $\Lambda \propto 1/t^2$ i.e. $\Lambda \propto t^{-2}$ as expected which resembles with the work investigated by Bertolami [11]. The gravitational constants is always positive and increases with cosmic time.Due to the positive pressure in the derived model there is no chance of dark energy. Hence the model is fully occupied with only barotropic fluid. Also, The solutions correspond to a Big-bangsingular model at point $t_s = -b/a$ where $a \neq 0$. At this point the kinematicalquantities such as Hubble's parameter, expansion scalar, shear scalar and mean anisotropy parameter all are diverge but the spatial volume and scale factor vanishes.

REFERENCES

[1] Perlmutter S. *et al.*, Astrophys. J. 517, 5 (1999)

[2] Riess A. G. *et al.*, Publ. Astron. Soc. Pacific (PASP) 112, 1284 (2000)

[3] Spergel D. N. *et al.*, Astrophys. J. Suppl. Ser. 148, 175 (2003)

[4] Peebles P. J. E., B. Ratra, Rev. Mod. Phys. 75, 559 (2003)

[5] Padmanabhan T., Phys. Rep. 380, 235 (2003)

[6] Vishwakarma, R. G.: Gen. Relativ. Gravit. 33, 1973 (2001)

[7] Cunha, J. V. and Santos, R. C.: Int. J. Mod. Phys. D 13, 1321 (2004)

[8] Carneiro, S. and Lima, J. A.: Int. J. Mod. Phys. A 20, 2465 (2005)

[9] Berman, M. S.: Gen. Rel. Grav. 23, 465 (1991)

[10] Kallingas, D., Wesson, P. S. and Everitt, C. W. F.: Gen. Relativ. Gravit. 24, 351 (1992)

[11] Bertolami, O.: NuovoCimento B 93, 36 (1986)

[12] Arbab, A. I.: Class. Quant. Grav. 20, 93 (2003)

[13] Sattar, A. and Vishwakarma, R. G.: Aust. J. Phys. 50, 893 (1997)

[14] Pradhan, A. and Yadav, V. K.: Int. J. Mod. Phys. D 11, 893 (2002)

[15] Pradhan, A., Pandey, P., Singh, G. P., Deshpandey, R. V.: Space. & Subs. 6, 116 (2005)

[16] Pradhan, A., Singh, A. K. and Otarod, S.: Roman. J. Phys. 52, 415 (2007)

[17] Singh, C. P. and Kumar, S.: Int. J. Mod. Phys. D 15, 419 (2006)

[18] Singh, C. P., Kumar, S. and Pradhan, A.: Class. Quant. Grav. 24, 455 (2007)